NAG Toolbox for MATLAB

g05ly

1   Purpose

g05ly sets up a reference vector and generates an array of pseudo-random numbers from a multivariate Normal distribution with mean vector a and covariance matrix C.

2   Syntax

```
[x, iseed, r, ifail] = g05ly(mode, xmu, c, n, igen, iseed, r, 'm', m,
'lr', lr)
```

3   Description

When the covariance matrix is nonsingular (i.e., strictly positive-definite), the distribution has probability density function

$$f(x) = \sqrt{\frac{\left|C^{-1}\right|}{(2\pi)^m}} \exp\left\{-(x - a)^{\mathrm{T}} C^{-1}(x - a)\right\}$$

where m is the number of dimensions, C is the covariance matrix, a is the vector of means and x is the vector of positions.

Covariance matrices are symmetric and positive semi-definite. Given such a matrix C, there exists a lower triangular matrix L such that $LL^{\mathrm{T}} = C$. L is not unique, if C is singular.

g05ly decomposes C to find such an L. It then stores m, a and L in the reference vector r which is used to generate a vector x of independent standard Normal pseudo-random numbers. It then returns the vector $a + Lx$, which has the required multivariate Normal distribution.

It should be noted that this function will work with a singular covariance matrix C, provided C is positive semi-definite, despite the fact that the above formula for the probability density function is not valid in that case. Wilkinson 1965 should be consulted if further information is required.

One of the initialization functions g05kb (for a repeatable sequence if computed sequentially) or g05kc (for a non-repeatable sequence) must be called prior to the first call to g05ly.

4   References

Knuth D E 1981 *The Art of Computer Programming (Volume 2)* (2nd Edition) Addison–Wesley

Wilkinson J H 1965 *The Algebraic Eigenvalue Problem* Oxford University Press, Oxford

5   Parameters

5.1   Compulsory Input Parameters

1:    **mode – int32 scalar**

Selects the operation to be performed:

mode = 0

Initialize and generate random numbers.

mode = 1

Initialize only (i.e., set up reference vector).

mode = 2

Generate random numbers using previously set up reference vector.

Constraint: $0 \leq \textbf{mode} \leq 2$.

2:    **xmu**(**m**) – **double array**

a, the vector of means of the distribution.

3:    **c**(**ldc,m**) – **double array**

ldc, the first dimension of the array, must be at least **m**.

The covariance matrix of the distribution. Only the upper triangle need be set.

Constraint: **c** must be positive semi-definite to ***machine precision***

4:    **n** – **int32 scalar**

n, the number of random variates required.

Constraint: $\textbf{n} \geq 1$.

5:    **igen** – **int32 scalar**

Must contain the identification number for the generator to be used to return a pseudo-random number and should remain unchanged following initialization by a prior call to g05kb or g05kc.

6:    **iseed**(**4**) – **int32 array**

Contains values which define the current state of the selected generator.

7:    **r**(**lr**) – **double array**

If **mode** = 2, the reference vector as set up by g05ly in a previous call with **mode** = 0 or 1.

5.2   Optional Input Parameters

1:    **m** – **int32 scalar**

Default: The dimension of the arrays **c**, **x**. (An error is raised if these dimensions are not equal.)

m, the number of dimensions of the distribution.

Constraint: $\textbf{m} > 0$.

2:    **lr** – **int32 scalar**

Default: The dimension of the array **r**.

If **mode** = 2, it must be the same as the value of **lr** specified in the prior call to g05ly with **mode** = 0 or 1.

Constraint: $\textbf{lr} > \textbf{m}(\textbf{m} + 1)$.

5.3   Input Parameters Omitted from the MATLAB Interface

ldc, ldx

5.4 Output Parameters

1: **x**(**ldx,m**) **– double array**

The array of pseudo-random multivariate Normal vectors generated by the function, with $X(i,j)$ holding the jth dimension for the ith variate.

2: **iseed**(**4**) **– int32 array**

Contains updated values defining the new state of the selected generator.

3: **r**(**lr**) **– double array**

If **mode** $= 0$ or 1, the reference vector that can be used in subsequent calls to g05ly with **mode** $= 2$.

4: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **mode** $\neq 0$, 1 or 2.

ifail $= 2$

On entry, **m** < 1.

ifail $= 4$

The covariance matrix **c** is not positive semi-definite to ***machine precision***.

ifail $= 5$

On entry, **ldc** $<$ **m**.

ifail $= 6$

On entry, **n** < 1.

ifail $= 8$

On entry, **ldx** $<$ **n**.

ifail $= 9$

On entry, invalid value for **igen**. **igen** must be the same as the value as specified in the prior call to g05ly with **mode** $= 0$ or 1.

ifail $= 11$

The reference vector **r** has been corrupted or **m** has changed since **r** was set up in a previous call with **mode** $= 0$ or 1.

ifail $= 12$

On entry, **lr** $<$ **m**(**m** $+ 1$).

7 Accuracy

The maximum absolute error in LL^T, and hence in the covariance matrix of the resulting vectors, is less than $(m\epsilon + (m+3)\epsilon/2)$ times the maximum element of C, where ϵ is the **machine precision**. Under normal circumstances, the above will be small compared to sampling error.

8 Further Comments

The time taken by g05ly is of order nm^3.

It is recommended that the diagonal elements of C should not differ too widely in order of magnitude. This may be achieved by scaling the variables if necessary. The actual matrix decomposed is $C + E = LL^T$, where E is a diagonal matrix with small positive diagonal elements. This ensures that, even when C is singular, or nearly singular, the Cholesky Factor L corresponds to a positive-definite covariance matrix that agrees with C within **machine precision**.

9 Example

```
mode = int32(0);
xmu = [1;
      2;
      -3;
      0];
c = [1.69, 0.39, -1.86, 0.07000000000000001;
     0, 98.01000000000001, -7.07, -0.71;
     0, 0, 11.56, 0.03;
     0, 0, 0, 0.01];
n = int32(10);
igen = int32(1);
iseed = [int32(1762543);
     int32(9324783);
     int32(42344);
     int32(742355)];
r = [0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0];
```

```
[x, iseedOut, rOut, ifail] = g05ly(mode, xmu, c, n, igen, iseed, r)

x =
    3.7228    -7.4911    -7.9865     0.1579
    0.1335     7.2616    -2.7775    -0.1209
    0.9254    12.2770     0.8045    -0.0237
   -0.7430     1.2303    -1.0565    -0.0646
    0.6148   -15.3318     1.3101     0.0812
    1.1606    20.4061    -3.6213    -0.0935
   -0.0647    -2.0038     2.9313     0.0035
    1.8238     6.4318   -10.0483    -0.0348
    2.6441   -12.3753    -2.4718     0.1373
   -0.0049   -19.3144     1.1728     0.1233
iseedOut =
    12448879
    13032530
    13959882
     2589587
rOut =
    4.5000
    1.3000
    0.3000
   -1.4308
    0.0538
         0
    9.8955
   -0.6711
   -0.0734
         0
         0
    3.0104
    0.0192
         0
         0
         0
    0.0367
    1.0000
    2.0000
   -3.0000
         0
         0
         0
         0
         0
         0
         0
         0
         0
         0
         0
ifail =
         0
```